Eights-on-Pylons

Where Does the Altitude Formula Come From?

From the FAA's Airplane Flying Handbook:

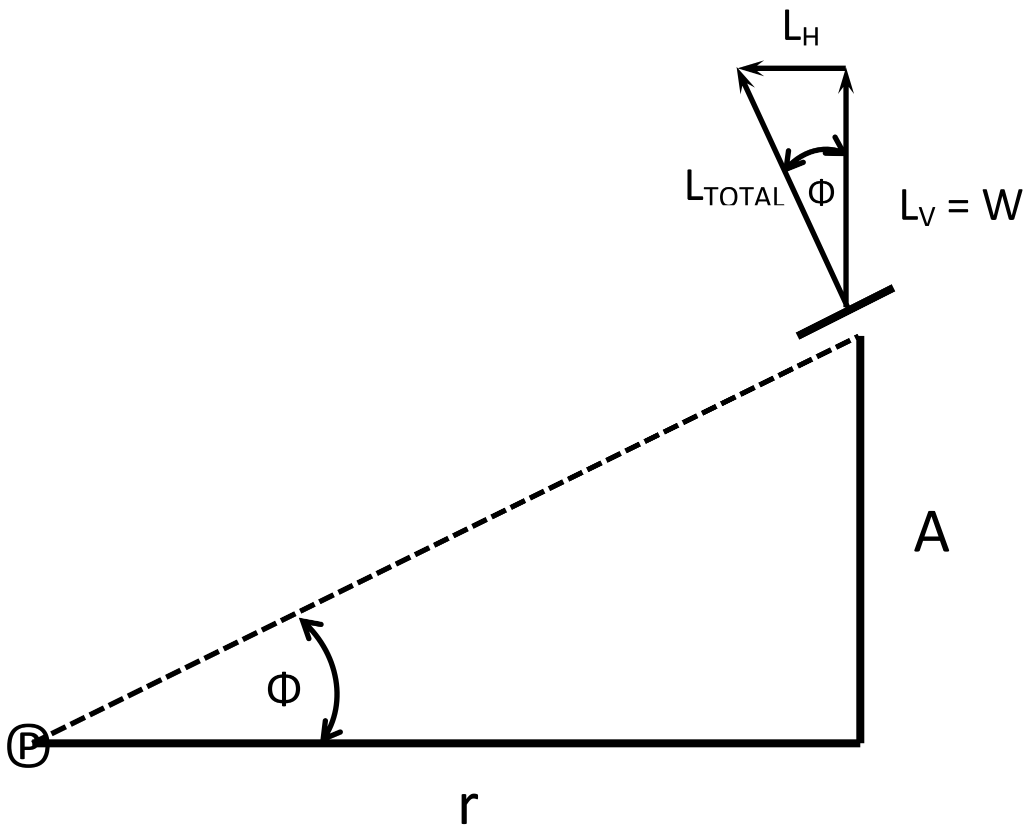
The eights-on-pylons is the most advanced and difficult of the ground reference maneuvers. Because of the techniques involved, the eights-on-pylons are unmatched for developing intuitive control of the airplane. Similar to eights around pylons except altitude is varied to maintain a specific visual reference to the pivot points. The goal of the eights-on-pylons is to have an imaginary line that extends from the pilot's eyes to the pylon. This line must be imagined to always be parallel to the airplane's lateral axis. Along this line, the airplane appears to pivot as it turns around the pylon. In other words, if a taut string extended from the airplane to the pylon, the string would remain parallel to lateral axis as the airplane turned around the pylon. At no time should the string be at an angle to the lateral axis.

They further say …

The altitude that is appropriate for eights-on-pylons is called the "pivotal altitude" and is determined by the airplane's groundspeed … a good rule of thumb for estimating the pivotal altitude is to square the groundspeed, then divide by 15 (if the groundspeed is in miles per hour) or divide by 11.3 (if the groundspeed is in knots), and then add the mean sea level (MSL) altitude of the ground reference. The pivotal altitude is the altitude at which, for a given groundspeed, the projection of the visual reference line to the pylon appears to pivot.

So, where does the formula for the pivotal altitude come from? Picture the pylon or pivot point, because usually there is no actual pylon, just a clearly identifiable spot on the ground (intersections of roads or power or communication towers – see the caveat later - appropriately spaced so as to have about four seconds or so of level flight between the two halves of the "8"). In the diagram below, the dotted line is the 'line of sight' to the pylon (pivot point) and the short line above the vertical line marked 'A' represents a line parallel to the lateral axis of the aircraft where you use a certain distance below the wing tip of our (high-wing, for ease of illustration) aircraft (picture a Cessna 172) that puts your eye line (the dotted line) parallel to the lateral axis. For a Cessna 172 this is a distance of about the height of a soda can below the wing tip or maybe a "tall boy" can depending on how high you sit in the seat. This would not be a line parallel to the bottom of the wing because the wing's shape, wash-out and dihedral would likely not make the line of the wing one that is parallel to the aircraft's lateral axis.

And, if you do choose a pair of towers as your pivot points be sure to use the top of the tower as the pivot point, not the bottom, since you want to have as much clearance above the tower as you can get. All-in-all it is probably less likely that you would find two towers that are very close to the same height spaced properly and oriented (with respect to the wind) properly vs. two spots on the obstruction-free surface and it seems that a surface-reference pivot point would be a safer choice. (See AFM 6-14 through 6-18 for specifics on how eights-on-pylons are flown). So, let's do the math:



Where:

℗ is the pylon / pivot point

Φ is the view angle parallel to the aircraft's lateral axis

r is the radius of the turn (you'll see what happens to this later)

A is the pivotal altitude

L_V is the vertical component of lift (assumed equal to the weight of the aircraft)

L_H is the horizontal component of lift

Also note that by the principles of geometry Φ is also the bank angle of the aircraft's longitudinal axis and here is assumed to be the angle from vertical of the total lift (L_{TOTAL}) vector.

For a coordinated turn of radius 'r' the horizontal component of lift is equal to the centrifugal force (F_C), where F_C equals the mass of the aircraft x centrifugal acceleration and the horizontal component of lift equals the weight of the aircraft x tan Φ. Since tan Φ = L_H / L_V solving for L_H gives L_H = L_V x tan Φ and L_V is assumed equal to the weight of the aircraft or the mass of the aircraft x acceleration of gravity.

So:

$$F_C = \frac{m}{g_c} * \frac{V^2}{r} = L_H = m\frac{g}{g_c} * \tan\Phi \quad \text{or} \quad \frac{m}{g_c} * \frac{V^2}{r} = m\frac{g}{g_c} * \tan\Phi$$

where V is the ground speed (we use ground speed because this is in reference to a fixed location on the ground). The asterisk (*) is used as the multiplication sign

You can see that $\frac{m}{g_c}$ appears on both sides of the equation so this reduces the second equation to:

$\frac{V^2}{r} = g * tan\Phi$ and solving for tan ϕ we get $tan\Phi = \frac{V^2}{r * g}$ (The $\frac{m}{g_c}$ cancel each other)

Looking again at the diagram we can also see that $tan\Phi = \frac{A}{r}$ so since $tan\Phi = \frac{V^2}{r * g}$ then $\frac{A}{r} = \frac{V^2}{r * g}$

You can also see that the r's cancel each other. Solving for A (the pivotal altitude):

$A = \frac{V^2}{g}$ .. so far so good but since the acceleration of gravity g is in feet per second squared (g = 32.174 ft/sec^2) and V is our ground speed in knots (nautical miles per hour) we have to throw in some conversion factors. To convert ground speed to feet per second we multiply V in NM/hr. x 6076 ft/NM divided by 3600 seconds per hour which gives us V x 1.6877 to get V converted to feet per second. But, since V is squared we have to square the conversion factor also.

$A = \frac{(V * 1.6877)^2\ ft^2/sec^2}{32.174\ ft/sec^2}$ where you can see that all the units cancel except for ft. in the numerator so you get altitude (above ground level) in feet. Completing the math you get

$$A = \frac{(V * 1.6877)^2\ ft}{32.174} = V^2 * \frac{2.8486}{32.174} = \frac{V^2}{11.295}$$ or with the denominator rounded up,

$A = \frac{V^2}{11.3}$ If you re-read the FAA's statement in the AFM, above, they call this a "good rule of thumb" but it is not a rule of thumb, it is the actual math with the denominator expressed to three significant digits.

For ground speed in MPH the conversion factor for speed is $(5280/3600)^2$ = 2.1511 which makes the denominator 32.174 / 2.1511 = 14.956, which rounded up to 3 significant digits is equal to 15.0

QED (which no, does NOT stand for 'quite enough, Dan!')

g_c is the gravitational constant 32.174 $\frac{lb_m - ft}{lb_f - sec^2}$ and is required any time you use mass measured in pounds mass (lb_m) and distance measured in feet. In this case the g_c on each side of the equation cancel each other but sometimes this is not the case. We are used to thinking of weight which is pounds force (lb_f) as the same as mass because the ***numerical*** value of the acceleration of gravity is the same as the gravitational constant. But, consider a 175 lb_m person on the moon who happens to weight 175 lb_f on earth. Since the acceleration of gravity on the moon is only 5.33 ft/sec^2 on the moon our 175 lb_m person weights (exerts a force of) only: $175\ lb_m x\ (5.33 \frac{ft}{sec^2}\ /32.174\ \frac{lb_m - ft}{lb_f - sec^2})$ = 29.0 lb_f. Those who use the metric system with mass in kilograms and distance in meters to find force in Newtons do not need a g_c.