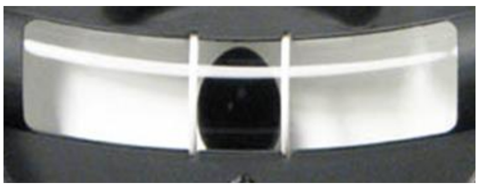
The Most Elegant Aircraft Instrument?

Daniel Sullivan December 2, 2020

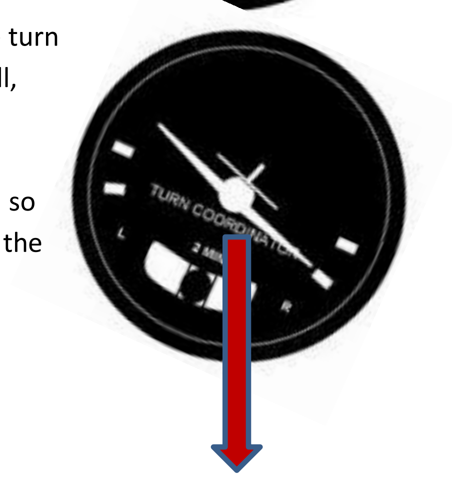
In science ‘elegant’ describes something that provides information in the simplest, most understandable way. So – I present as a candidate for the most elegant aircraft instrument: The inclinometer (AKA slip indicator, slip/skid indicator, slip ball, slip/skid ball). Not the entire instrument shown at the right (a turn and bank indicator with inclinometer – and just below that, the even more sophisticated turn coordinator with inclinometer). Nope – just the glass tube filled with oil with a metal ball that can move from side to side. This part, below:



Pure elegance. So simple – requires no power, always works, always gives the correct information.

We’ll use the version above since it provides a white arc near the top of the tube which as you will see will be handy in part of our “proof” of the elegance of this instrument.

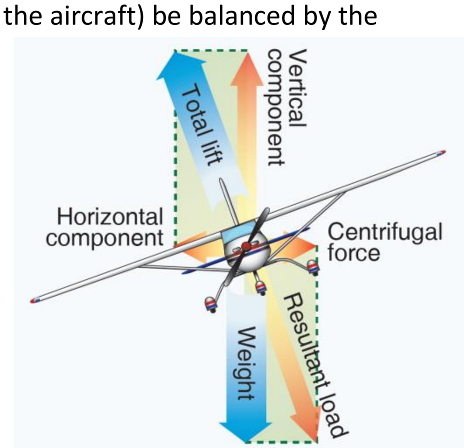
Our turn coordinator will help with one point: Although our brains tend to make the view of the turn coordinator (and, so along with it, the inclinometer) appear as it does at the right, with respect to the rest of the world (in this case, with the ground or more accurately the center of the earth at least from a gravitational standpoint) in a turn the turn coordinator is at an angle like the one below the first one shown. Straight down is, well, straight down.



With respect to the earth your aircraft flying at 150 kTAS is banked at a 22° angle – and so therefore is the turn coordinator and, to the point of this whole discussion, specifically the inclinometer itself.

And the reason that even though the inclinometer is tilted at a 22° angle in this example the ball is centered in the tube is … well, the whole point that is about to be made.

So on to the rest of the story.
A coordinated turn requires that the horizontal component of lift (the force that turns the aircraft) be balanced by the centrifugal force created by the aircraft turning in an arc (or, if continued long enough, in a circle). You are probably familiar with the FAA’s version of this, as shown by the figure at the right. These are the forces acting on the aircraft –

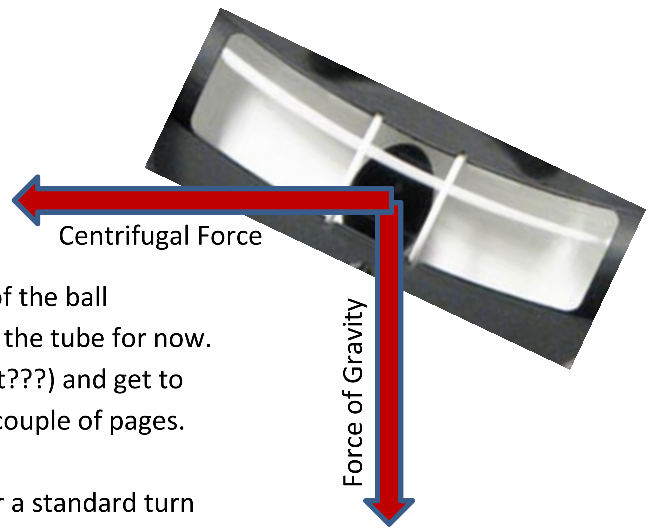


The vertical forces – the vertical component of lift and the weight of the aircraft (although that is not really correct, the vertical component of lift equals the weight of the aircraft plus any other downforce such as that exerted by the tail that keeps the plane level when the center of lift is behind the center of gravity); and …

The horizontal forces – the horizontal component of lift and the centrifugal force (which is an inertial force caused by the rotation). The upward and downward vertical forces are equal in a level or constant rate ascending or descending turn (yes, even if ascending or descending if the rate is constant the forces are constant – if you don't understand that ask your local Physicist or Engineer). The horizontal forces are equal (and opposite, so they cancel) in a coordinated turn – you are following the path of the circumference of the turn circle, otherwise you would be slipping or skidding.

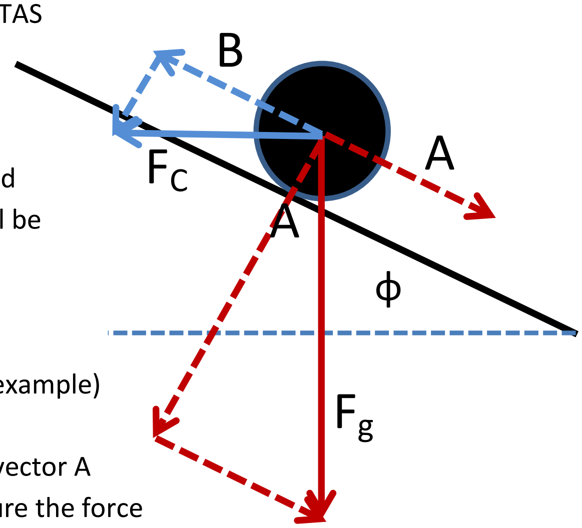
BUT – the forces that act on the BALL of the inclinometer are – Centrifugal force and gravity. OK, gravity is an acceleration but you'll see that centrifugal force is the mass of the ball times the centrifugal acceleration and the force on the ball due to gravity is the mass of the ball times the acceleration of gravity and that the mass of the ball is cancelled out in the equations we will derive in a moment.

Taking a look at the inclinometer with respect to the earth (keeping with our 22° bank angle example) the inclinometer looks like this and the forces acting on the ball are as shown →



To simplify the image we'll use line drawings to represent the position of the ball on the bottom surface of the inclinometer tube, neglecting the curve of the tube for now. And, if you want to skip the math (Really??? Who would want to do that???) and get to the lessons that this elegant instrument can teach us then skip down a couple of pages.

The inclinometer is at our 22° angle which is the bank angle required for a standard turn at 150 kTAS (using the rule of thumb: Bank Angle (std turn) = 15% of the TAS (10% + half of 10%, so 15 + 7.5 = 22.5° or roughly 22°).
An approximate angle (but very close) – to be refined.



The force due to gravity is: $F_g = mg/g_C$ where m is the mass of the ball and g_C is the gravitational constant (required, since pounds mass and feet will be the units used here) although both m and g_C will cancel out.

The component of the force of gravity held up by the bottom of the inclinometer tube is $F_g \cos(\phi)$ where ϕ is the bank angle (22° in our example) and – the important one – the component of the force of gravity pulling the ball down the inclinometer is $F_g \sin(\phi) = mg/g_C \sin(\phi)$ = A (the force vector A is shown in two places so you can see how it relates to F_g. and also picture the force working through the center of the ball, opposing force B. The component of centrifugal force (F_c) that pushes the ball up the inclinometer tube (opposing force A) is force B = $F_c \cos(\phi)$, where $F_c = m/g_C \frac{TAS^2}{R} \cos(\Phi)$ where TAS = aircraft true airspeed and R is the radius of a standard two-minute turn (which we will calculate in a moment).

So for a coordinated turn, the ball motionless in the center of the inclinometer, force A must equal force B, so …

$mg/g_C \sin(\phi) = m/g_C \frac{TAS^2}{R} \cos(\Phi)$ – and with m/g_C on both sides of the equation they cancel, leaving:

$g\sin(\phi) = \frac{TAS^2}{R} \cos(\Phi)$ so this expression turns out to be (having removed the mass term) the component of acceleration due to gravity along (down the slope of) the inclinometer tube is equal to the component of acceleration due to centrifugal force along (up the slope of) the inclinometer tube.

So what is R for a two minute turn at 150 kTAS? Easily derived from the formula for the circumference of a circle C = 2πR, so R = C/2π. Since we are solving for a R of a standard 2-minute turn and at 150 kTAS (15,190 ft/min) the distance traveled (the circumference of the circle) in two minutes is 30,380 ft so: R = 30,380/2π = 4,835 feet.
Rule of thumb: R (std turn) = ½ TAS/100 + 0.05 (in NM) = ½ (150)/100 + 0.05 = 0.75 + 0.05 = 0.80 NM (4,861 ft – 0.5% off of exact).

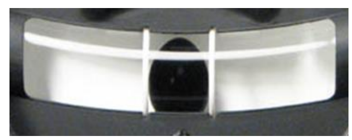
And the bank angle for a standard 2-minue turn at 150 kTAS? Well, that involves some really fun math that won't be covered here but it comes down to: $\tan \Phi = \frac{V^2}{gR} = \frac{V^2}{\left(32.174\, x \frac{V}{188.5}\, x \frac{12{,}960{,}000}{6076.12}\right)} = \frac{V\,(kTAS)}{364.1}$ so for V = 150 knots, we get $\Phi = \arctan\left(\frac{150}{364.1}\right) = \arctan(0.412)$ = 22.4° (pretty close to our original approximation of 22° - or actually very close to the rule of thumb value of 22.5° - 22.4° is used for mathematical accuracy in the proof but the fraction of a degree means little in practice – no one holds a turn to within a fraction of a degree of bank angle).

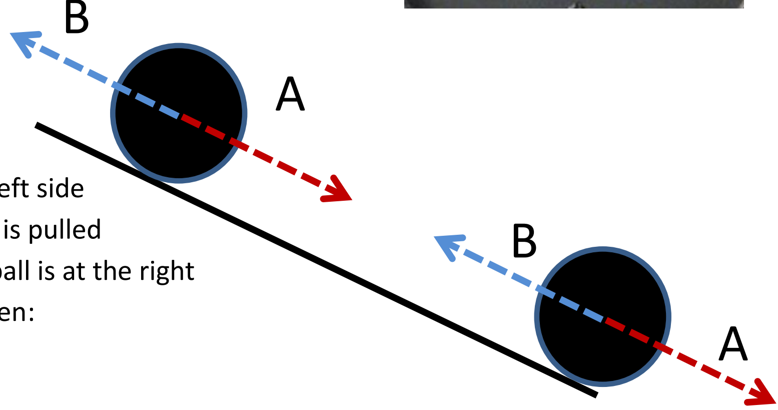
Do the accelerations (and therefore the forces) balance?

A = 32.174 ft/sec^2 x sin(22.4°) = 12.26 ft/sec^2 and

B = ($\frac{150}{3600}$ x 6076)^2 ft^2/sec^2 / 4835 ft) x cos(22.4°) = 13.256 x cos(22.4°) = 12.26 ft/sec^2 Q.E.D., as the math folks say.
(note the conversion factors for hours to seconds – 3600, and feet per nautical mile – 6076)

But why the curve in the tube? Well, if you consider that if the ball could move somewhere along the tube if you were not perfectly coordinated at all times (which is usually the case) then when your turn became perfectly coordinated the forces would equal and the ball would stop – wherever it was in the tube. Centered or not; no (net) force, no movement (or, no stopping of any existing movement – at least until the ball hits the end of the tube).
But if the tube curves from the center – maybe 5° or so like the one in the original picture then if the ball is up to the far left, until the flat (center) part of the tube is reached the forces are:





A = g sin(22.4 + 5) = 32.174 x sin(27.4) = 14.81 and
B = 13.256 x cos(22.4 + 5) = 13.256 x cos(27.4) = 11.77

Force (acceleration) A is greater than B when the ball is on the left side of the tube (+5°). When the turn becomes coordinated the ball is pulled downward until the forces equal at the center. Likewise, if the ball is at the right side of the tube (where the angle is the bank angle minus 5°) then:

A = g sin(22.4 - 5) = 32.174 x sin(17.4) = 9.62 and
B = 13.256 x cos(22.4 - 5) = 13.256 x cos(17.4) = 12.65
so centrifugal force pulls the ball upward (up our tilted inclinometer tube) until the ball centers.

All in all a pretty clever instrument. Elegant, you might say. So what have we learned from this "magic ball"?

What the Ball tells us ….

1. Since the Ball responds only to gravity and centrifugal force, then if the Ball is towards the low wing ("falling down the tube") then gravity is greater than centrifugal force – so you need more centrifugal force … or, what the Ball is telling you is that you need to tighten your turn. Remember that centrifugal acceleration is V^2/r so adding rudder makes r smaller, increasing centrifugal acceleration (and, when applied to the mass of the Ball, increasing centrifugal force on the Ball). No, the aircraft is not falling – just the Ball, because the aircraft is tilted (rolled) to one side and so is the inclinometer tube. Gravity (inadequately opposed by centrifugal acceleration) moves the Ball down towards the lowered wing.

2. And, if the Ball is towards the high wing you need less centrifugal force, so you need to reduce the amount of rudder to increase the turn radius (make 'r' bigger) reducing centrifugal force and allowing the Ball to fall towards the middle of the tube.

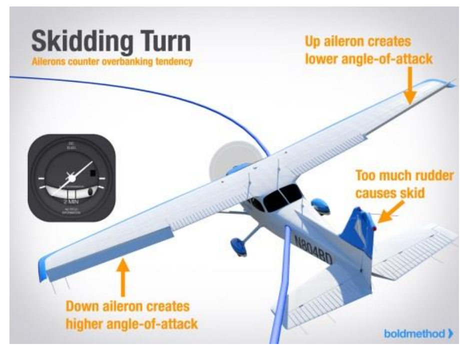
3. The Ball agrees that for situation #1 that "step on the ball" makes a certain degree of sense. Step more (on the rudder) when the ball is "low" in the tube. But the Ball does not agree that "step on the ball" makes immediate sense for situation #2. The Ball thinks this should be "ease off of the ball" (ease off of the rudder in the direction of the turn) – don't step on the opposite rudder which would not be the right thing to do if the goal was to get the aircraft into a coordinated turn. Think of the ball as needing to be stepped on more (if low in the tube, to push it up some) or less (if high in the tube, to let it fall down some). The Ball wants to make a new phrase – 'Use your foot to position the ball – Push more to move it up, push less to let it move down'.

4. In a coordinated turn, the Ball sits in the middle of the inclinometer tube. It does so because at that point the net force on the Ball is pointing in a direction parallel to the vertical axis of the aircraft – straight down to the floor of the aircraft. And, if the net force of the turn is pushing the Ball straight down to the floor then it must be pushing everything else in the aircraft straight down to the floor also – including the seat of your pants. And, including the flashlight suspended by a string from the whiskey compass. So the Ball gives you the answer to the question: If your attitude indicator fails can you tie a flashlight to your compass with a string and judge the bank angle comparing the string angle to the center support of the windscreen? The Ball says the answer is no – the net force on the flashlight in a coordinated turn is, just like the Ball, straight down (parallel to the vertical axis of the aircraft) to the floor so the flashlight always hangs straight with respect to the vertical axis (and, straight with respect to the vertical center windscreen support bar). If you look back on page 1 there are two solid lines – the red one, the force due to acceleration of gravity and the blue one – the force due to centrifugal acceleration. In your mind, move the tail of the blue arrow to the head of the red arrow the head of the blue arrow (the vector sum of the two forces) will lie straight down along an extension of the (dotted) line which is the component of the gravitational force perpendicular to (pointing down to the floor, parallel to the vertical axis of the aircraft) a line tangent to the bottom of the inclinometer tube.

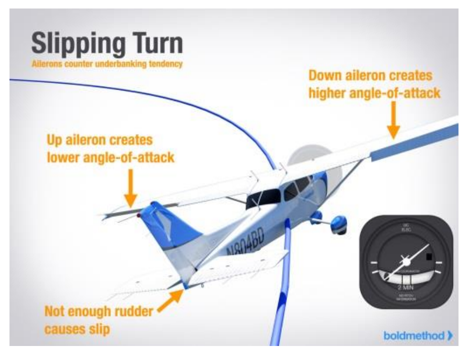
5. Uncoordinated turns – slipping, skidding … terms that are often used but how well are they understood? The Ball is here to help. Sure, we have the 'slip in, skid out' mnemonic to help you answer the questions on the written exam about slips and skids but what is really happening? I for one find the terms less helpful than if Goldilocks had been the one to define terms – Too tight, not tight enough, just right. But for better or worse we're stuck with slip and skid. Not that they are altogether inaccurate – in fact they do make sense if you think about a slip or a skid if you were in contact with the ground.

You're walking on ice and try to turn – you slip … continuing forward, not turning. Or, you're driving your car and you turn too tight so the rear wheels lose their grip and the back end starts to move in the direction away from the turn (the back end starts 'coming around') and your car is now in a skid. You might also compare the skidding of a car – a loose rear end and a slip in a car – on ice, where you turn the wheel but the car does not turn. The car continues straight like you do when you are walking on ice and try to turn but you slip and continue forward (probably on your butt), not turning or at least not turning enough. In the car – when you skid the back end goes outside the turn (the turn is too tight), so in a right turn the rear end is left of the (desired) path of the turn. In a slip, the back end of the car does not turn (the turn is not tight enough) and you continue straight, so in an attempted right turn with a slip the back end of the car stays inside of the desired path of the turn.

The Ball shows you where the tail of the aircraft is with respect to where it should be if it is not in the center. With the help of a couple of illustrations from **boldmethod** (an excellent web site for aviation information and knowledge) we'll illustrate this.



Banked left, the little airplane of the turn coordinator tilted left; if the Ball is to the right it is telling you that the tail is to the right of where it should be – outside of the turn. A skid. Too much (left) rudder. Lift you left foot a bit to let the Ball move down (along the slant of the wings of the little aircraft of the turn coordinator) until it is in the middle.



If the Ball is to the left it is telling you that the tail is to the left of where it should be – inside the turn. A slip. Not enough (left) rudder. Push on your left foot more to push the Ball up the slope of the wings of the little aircraft of the turn coordinator.

Skids are more dangerous than slips. Read why on the boldmethod site **Why Skids are More Dangerous than Slips**.

6. Since gravity is at work, if you don't want to turn at all when you are banked then the Ball will let you know you are moving straight ahead if it is off-center by the amount of the angle of your bank. A handy thing for multi-engine aircraft with one engine inoperative. You bank towards the engine that is running – usually four degrees or so in your average light twin. If you are flying straight (no turn) the center of the ball will be on or just a bit less beyond the left or right vertical line that defines the centered area on the side of the operative engine. (About half of the ball 'outside the cage'). Ask your local Physicist or Engineer if you want to know why you should fly with the ball in this position if your twin engine airplane suddenly becomes a lopsided Cessna 172.

7. All this elegance does not mean it comes at a high price. An inclinometer (even with all the bureaucratic laa-de-daa required to meet the TSO requirements): About $100. The satellite system to make your GPS work (and not even counting the cost of yours and everyone else's GPS receiver itself): $12 billion dollars to put into place and nearly a billion dollars a year to maintain.

And – Do you really think it is a coincidence that an ***elegant*** lady wears a ***Ball*** gown? Pure elegance.