Turns and the Overbanking Tendency

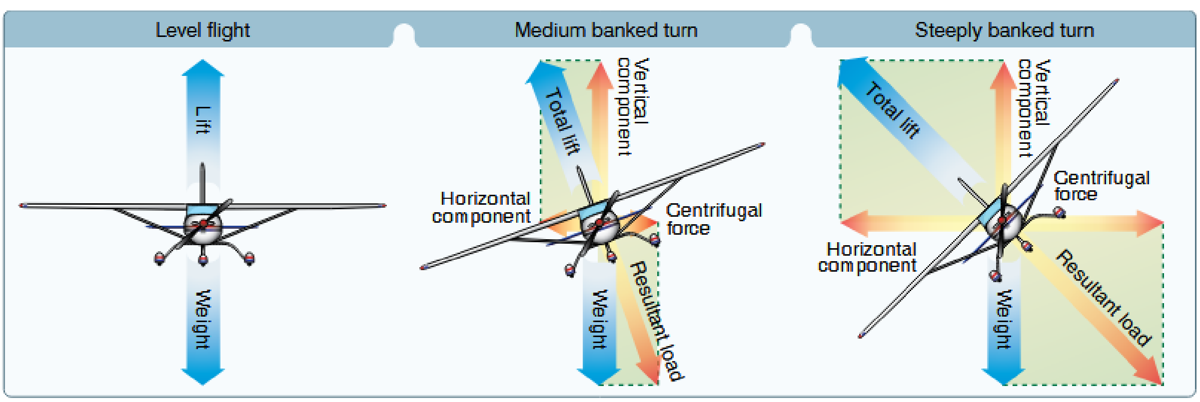
Daniel Sullivan November 3, 2021

Turns – seems simple enough. Birds do it. Bees do it. I suppose even fleas, educated or not, do it. So what's the big deal? Well, a discussion on the **Professional Pilots Rumor Network (PPRuNe)** (a spot to find lots of great information and discussions and to share knowledge or just your opinion) regarding **Overbanking Tendency** prompted me to do an overview on the topic. Seems to be a bit of interest in what happens when airplanes bank (and turn) based on their discussion so hopefully this might help make sense of the whole thing.

Hand-Waving and Paper Towel Drawings

Before we get into the math I'll present some of the basics. Stuff you probably already know but hopefully this will set the stage for what is to come.

1. An aircraft is not a boat. It turns because of a horizontal force (created by tilting the lift vector – banking the aircraft). The rudder gets the nose pointed in the right direction – coordinating the turn, so that the longitudinal axis of the airplane is pointing in the same direction as the direction of the turn, or at least tangent to the arc of the turn.



It is the horizontal component of lift that causes the aircraft to turn, not the rudder.

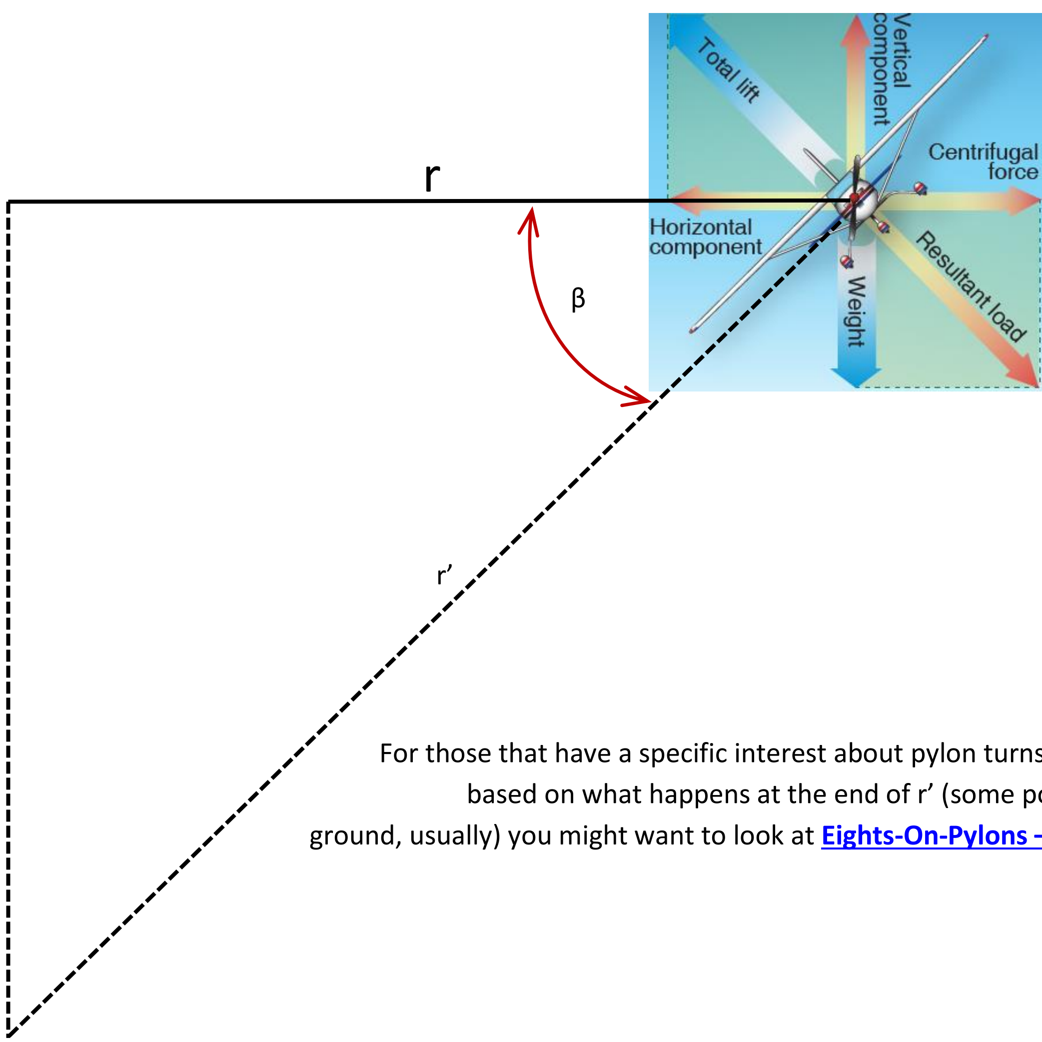
2. Unless stated otherwise we are talking about coordinated turns – turns with no slip or skid.

3. This discussion assumes that all the lift is provided by the wings. Not true even for the simplest of aircraft but for the most part close enough when considering your typical general aviation airplane. We are clearly not talking about the kinds of turns (and the physics thereof) of high-performance aircraft that can do knife-edge flying because the fuselage and somewhat up-tilted nose / down-tilted tail provides a component of vertical thrust from the engine that provides the lift (stunt planes with high-power engines can do this too).

So, the horizontal and vertical components of lift, as pictured above, comes only from the total lift vector produced by the wing. As such they are dependent on total lift and therefore dependent on each other through their relation to the bank angle.

4. Contrary to common teaching, and the left most (level flight) illustration above, lift is not equal to weight. It is equal to the total downward force which is aircraft weight + tail downforce. Since for any given circumstance the tail downforce component is a constant proportion of weight (center of gravity does not change significantly, if at all, for the time required to fly a given turn) what is used in the calculations ***is*** lift = weight but this could easily be some small factor (5 or 10% more) that is kept constant to make things a little more precise but it will not change the principles to be developed – That is, it will not change what the formulas we derive next "tell" us.

5. The derivations that follow use a frame of reference of a turn around a point that is horizontal (with respect to the earth, or more correctly, perpendicular to the weight vector) a given distance (the turn radius) from the center of gravity of the aircraft (which is where all components of lift, weight and centrifugal force works through). You could also derive the equations using a conical shape (a turn around a point located along a line parallel to the lateral axis of the aircraft, extending from the center of gravity of the aircraft). You can use either – you end up with the same equations (if you would like that demonstrated please ask your local Physicist or Engineer). So, when we talk about the radius of a turn we are referring to r, not r′. The bank angle is β. Keep this diagram in mind – it will be the basis of much of what comes next.



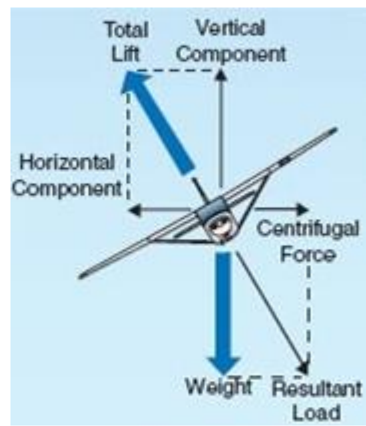
For those that have a specific interest about pylon turns which are based on what happens at the end of r′ (some point on the ground, usually) you might want to look at **Eights-On-Pylons – the Math**

6. Velocity is true airspeed. Although indicated airspeed works for things like lift, optimum or maximum speed limited by pressure and other performance parameters the physics of motion of a mass (the aircraft) is governed by the actual (true) airspeed. Yes, if motion is with respect to a fixed object on the ground that motion may be referenced to (in essence, related to or dependent on) ground speed but that is just true airspeed corrected for wind with the motion of the aircraft expressed as relative to the ground. For the purposes of this analysis motion with respect to the ground will not be a consideration so velocity will be the true airspeed of the aircraft.

7. Although it probably goes without saying, a standard turn is the usual standard turn for GA aircraft – 3° per second or a 'two minute turn'.

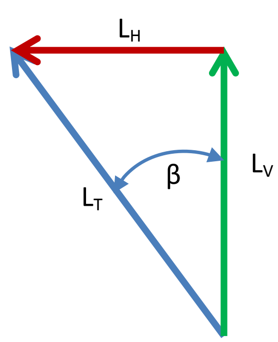
   Now for some math

Basic Math of a Turn



In the diagram on the right the aircraft is shown in a coordinated turn – How do we know? Because the horizontal component of lift is equal to the opposing centrigugal force. Equal and opposite, these cancel, so no net force that would cause you to move towards one side or the other (so, no slip or skid). *And – feel free to skip anything, below, that you already know.*

The vertical component of lift is equal to the aircraft's weight (as explained, and qualified in point #4, above). What we are interested in is the force that causes the aircraft to turn – the horizontal component (of lift). Re-drawing the triangle so that the horizontal component of lift is at the top will help make the geometry more clear:



The green arrow is the vertical component of lift (L_V)
The aircraft's weight is its mass (m) x acceleration of gravity (g), so
$L_V = m\frac{g}{g_C}$
(g_C is the gravitational constant needed when mass and acceleration are expressed in pounds mass (lb_m) and ft/sec^2 , respectively)

L_H (the horizontal component of lift) is equal to the total lift L_T times sin(β): $L_H = L_T \sin(\beta)$, *and*
$L_T = L_V$ divided by cos(β): $L_T = L_V/\cos(\beta)$, so since $L_H = L_T \sin(\beta)$, substituting for L_T we get:
$L_H = L_V \frac{\sin(\beta)}{\cos(\beta)}$ which is the same as: $L_H = L_V \tan(\beta)$ …
And since $L_V = m\frac{g}{g_C}$
$L_H = m\frac{g}{g_C} \tan(\beta)$
The horizontal component of lift is the weight of the aircraft times the tangent of the bank angle.

For a coordinated turn of radius r the horizontal component of lift (L_H) is equal to the centrifugal force (F_C), where F_C equals the mass of the aircraft times centrifugal acceleration: $F_C = \frac{mV^2}{rg_c}$. Since $F_C = L_H$

$$\frac{mV^2}{rg_c} = m\frac{g}{g_c}\tan(\beta)$$

and very conveniently the $\frac{m}{g_c}$ terms, which are on both sides of the equation, cancel out, leaving:

$\frac{V^2}{r} = g\tan(\beta)$ - rearranging this equation to solve for r gives:

$r = V^2/g\tan(\beta)$ (eqn. 1)

Standard, Coordinated Turn

For a standard turn you would travel a distance of V x 2 min. (where V is the TAS in NM/min). The circumference of a circle would equal that distance, so:

C = V x 2 min = 2πr so the radius of a standard turn is r = (V x 2 min)/2π.
Using the necessary conversion factors for V expressed as the TAS in knots (NM/hr) we get the radius of a standard turn (we'll call this r_{ST} to distinguish it from a turn of any value of r) for a given TAS:

$$r_{ST} = V\frac{NM}{hr}\,x\,\frac{1\,hr}{60\,min}\,x\,2\,min/2\pi = V\frac{NM}{hr}/188.5 \quad \text{(eqn. 2)}$$

Rearranging equation 1 to solve for the bank angle β gives:

$\beta = \arctan(V^2/g\,r)$

The problem here is that we like V to be in knots and r, at least some times, in nautical miles. So, we need to build in some conversion factors so we get them into the same units as g, the acceleration of gravity, which is 32.174 ft^2/sec^2. The units for V/r are $\frac{NM^2/hr^2}{NM}$ which reduces to NM/hr^2

$$NM/hr^2 \text{ x } \frac{\frac{6076\,ft}{NM}}{\frac{3600\,sec^2}{hr^2}} \text{ gives } 6076\,ft/12{,}960{,}000\,sec^2 = 1\,ft/2133\,sec^2$$

So for V in knots and r in NM,

$\beta = \arctan(\frac{1}{2133}V^2/g\,r)$ and for the case of a standard turn the bank angle for the standard turn is:

$\beta_{ST} = \arctan(\frac{1}{2133}V^2/g\,r_{ST})$ - substituting for r_{ST} using equation 2 and inserting the value for g we get:

$$\beta_{ST} = \arctan(\frac{188.5}{2133} \text{ x } V/g) = \arctan(\frac{188.5}{2133} \text{ x } V/32.174) = \arctan(V/364.1)$$

So What About the Bumble Bee?

Yes, the bumble bee does not know any of this and there is a myth that aerodynamically the bumble bee should not be able to fly but the bumble bee does not know that so goes ahead and flies anyway. Of course that's a myth. Bumble bees can fly. Maybe we are just not smart enough to understand how they do it.

But we are not 'natural fliers' like bees and birds. So maybe it is a good thing for us to understand at least to some degree how an airplane flies. And describing the physics and giving some of the math behind the actual performance might help that understanding.

You might also argue that no matter how smart you are you are not going to be able to divide something by 188.5 or calculate the arctangent of some equally difficult to calculate number in your head. And you are right. So –

Rules of Thumb to the Recue!

There are three rules of thumb (RoT) for the radius of a standard turn (r_{ST}):

1. r_{ST} = (½ TAS) / 100 (NM)
2. r_{ST} = (½ TAS) / 100 +0.05 (NM)
3. r_{ST} = (½ TAS) / 100 + 5% of the r_{ST} = (½ TAS) / 100 value (NM)
   So for this third method, if TAS is 120 knots then = (½ TAS) / 100 = 0.60 and 5% of the 0.60 is one half of one tenth or half of 0.06 which is 0.03, so r_{ST} = 0.60 + 0.03 = 0.63 NM

I use the second RoT – easy to do and fairly accurate, although the third version is not all that hard either. Here's how they compare to the actual math:

	Std 2 min. turn radius (NM)						
TAS (knots)	**Math**	**RoT #1**	**% Error**	**RoT #2**	**% Error**	**RoT #3**	**% Error**
80	0.42	0.40	-5.8%	0.45	6.0%	0.42	-1.0%
90	0.48	0.45	-5.8%	0.50	4.7%	0.47	-1.0%
100	0.53	0.50	-5.8%	0.55	3.7%	0.53	-1.0%
110	0.58	0.55	-5.8%	0.60	2.8%	0.58	-1.0%
120	0.64	0.60	-5.8%	0.65	2.1%	0.63	-1.0%
130	0.69	0.65	-5.8%	0.70	1.5%	0.68	-1.0%
140	0.74	0.70	-5.8%	0.75	1.0%	0.74	-1.0%
150	0.80	0.75	-5.8%	0.80	0.5%	0.79	-1.0%
160	0.85	0.80	-5.8%	0.85	0.1%	0.84	-1.0%
170	0.90	0.85	-5.8%	0.90	-0.2%	0.89	-1.0%
180	0.95	0.90	-5.8%	0.95	-0.5%	0.95	-1.0%

You could argue that any of the three rules of thumb yield results that are "close enough".

The rule of thumb for the bank angle required for a standard turn uses a mathematical rule that the tangent of (relatively) small angles is approximately equal to the angle (expressed in radians, so we convert that to degrees as shown below).

Since tan β for small angles $\cong \frac{\beta \ x \ \Pi}{180}$ then

$\tan\beta \cong \frac{\beta \ x \ \Pi}{180} = \frac{V \ (knots)}{364.1}$ Solving for β:

$Standard\ turn\ bank\ angle\ \beta_{ST} = V(knots) x \frac{180}{(346.1 \ x \ \Pi)} = V\ (knots)\ x\ 0.157$ where V is the TAS

Simplifying this:

$$Standard\ turn\ bank\ angle\ \beta_{ST} = 15\%\ of\ TAS\ (knots)$$

15% is relatively easy math – just take 10% of TAS, cut that in half and add that half to the 10%.
TAS = 140 knots, so β_{ST} = 10% of 140 (14) + half of 14 (7) = 21°
If you encounter a 0.5 just round up – no one can hold a bank angle within half a degree.

The rule of thumb is valid for speeds below about 180 knots for two reasons. First, the rule makes a assumption that holds only for smaller bank angles and therefore only for lower speeds. At speeds approaching 200 knots the mathematical error begins to increase and will eventually become excessive. Second, for conventional aircraft the bank angle is usually limited to 25 to 30 degrees so the rule of thumb is valid only about 180 knots TAS. You might trust it to about 200 knots but remember, the physics dictates that this is true airspeed so you might keep the 180 knot limit in mind to keep some boundaries on this.

The Rule of Thumb slightly underestimates the bank angle found using the actual math for slower speeds and slightly overestimates it for faster speeds. At 120 knots TAS the rule underestimates the bank angle by 1.3%. At 180 knots it overestimates the bank angle by 2.6%. At 140 knots the rule vs. calculated bank angle is almost the same, the rule of thumb underestimating the bank angle by less than 0.2%.

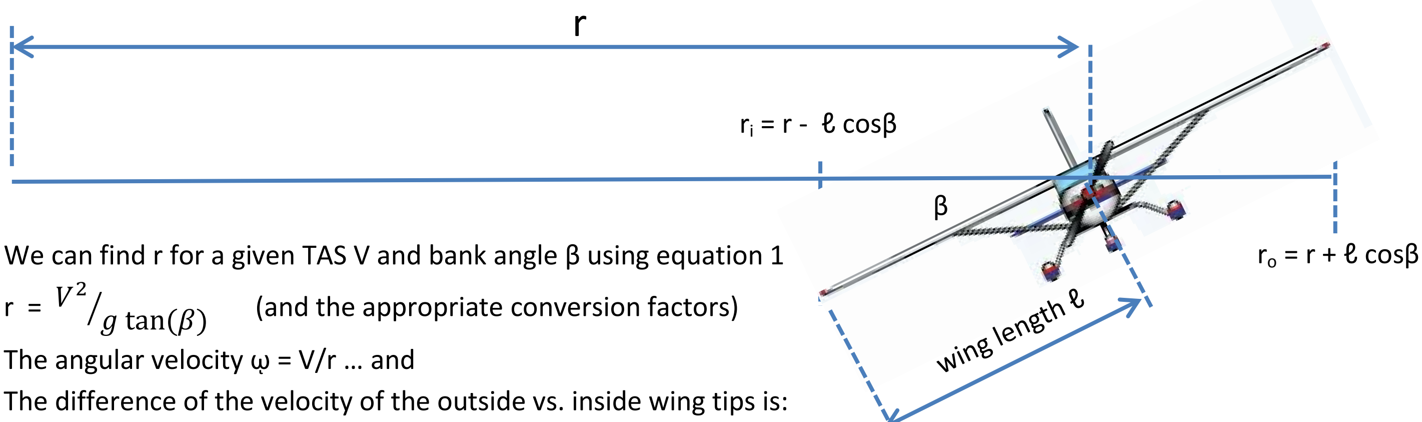
As noted above, this rule of thumb is not applicable to high speeds or high performance aircraft. A standard two minute turn at 420 knots would require a turning radius of 2.23 NM. Calculating the required bank angle we get a bank angle of 49.1°. Using the rule of thumb we get 420 x 0.15 = 63°, a significant error with a resulting 28% over-estimation of the required bank angle. But the error notwithstanding most commercial air carrier SOPs limit the allowable bank angle under normal conditions to 25° (some may allow up to 30°) for passenger comfort so turns at true airspeeds of 300 or 400 kTAS have a far wider radius (and lower rate) than a standard 2 minute turn. High-performance aircraft may have a turn coordinator calibrated to show a 4 minute turn. For those interested in turns at various speeds and bank angles you may want to check out the Airspeed / Bank Angle / Turn Radius Chart from the Code7700.com (an excellent source of information) page on Turn Performance.

The Overbanking Tendency

FAA publication FAA-H-8083-3C (current as of the writing of this article) states:

> *As the radius of the turn becomes smaller, a significant difference develops between the airspeed of the inside wing and the airspeed of the outside wing. The wing on the outside of the turn travels a longer path than the inside wing, yet both complete their respective paths in the same unit of time.*
>
> *Therefore, the outside wing travels at a faster airspeed than the inside wing and, as a result, it develops more lift. This creates an overbanking tendency that needs to be controlled by the use of opposite aileron when the desired bank angle is reached.*

Many have argued that there is no such thing as overbanking tendency but that is not true. It can be argued that the significance of (or maybe, the amount of work the pilot has to do to overcome) the overbanking tendency does vary between aircraft due to aircraft design. Most general aviation aircraft tend to be pretty stable (require little or no compensation for overbanking) in moderately banked turns. The trade-off is that the built-in stability in moderate turns tends to make the aircraft want to return to level flight in shallow bank turns and need some pilot-input compensation in steep turns. Add to that a particular aircraft's vertical location of the center of gravity and its pendulum effect, the short-tail effect, degree of spiral instability (or from another perspective, directional stability), conventional vertical rudder / horizontal tail vs. V-tail and a bunch of other things give each aircraft their own personality in a turn. Regardless, to one degree or another the speed of airflow over the wing differs between the inside and outside wing and so the potential for overbanking exists to some degree, at least at some bank angle and beyond. That really smart guy Peter Garrison wrote in the November 2011 *Flying Magazine*: **Is there an Overbanking Tendency?** Interesting reading.



We can find r for a given TAS V and bank angle β using equation 1

$r = V^2 / g\tan(\beta)$ (and the appropriate conversion factors)

The angular velocity $\omega = V/r$ … and

The difference of the velocity of the outside vs. inside wing tips is:

$V_o - V_i = \omega(r_o - r_i) = \omega \times 2\ell\cos\beta$; 2ℓ equals the total wing span(S) so this could be used here instead.

Lift is a function of V^2 so the ratio of lift between the outside and inside wingtip will be $\Delta L = (V_o/V_i)^2$

Using an aircraft with a wing span S of 36 feet (so ℓ is 18 ft.) at various airspeeds and bank angles we can calculate the difference in tip speed and lift generated at the wing tips. You could choose any point between the wing root and wing tip. It would not make a difference since provided the ratio of lift between the outside and inside wing is greater than zero there will be a net force that, acting though some lever arm will result in a rolling moment towards the lowered wing. For our aircraft with ℓ = 18 ft.

	Bank Angle β (degrees): 30				Bank Angle β (degrees): 45				Bank Angle β (degrees): 60			
TAS (knots)	r (feet)	ω (rad/min)	$V_o - V_i$ (ft/min)	ΔL	r (feet)	ω (rad/min)	$V_o - V_i$ (ft/min)	ΔL	r (feet)	ω (rad/min)	$V_o - V_i$ (ft/min)	ΔL
80	981	8.25	297	1.08	567	14.30	515	1.14	327	24.76	891	1.25
100	1534	6.60	238	1.05	885	11.44	412	1.08	511	19.81	713	1.15
120	2208	5.50	198	1.03	1275	9.53	343	1.06	736	16.51	594	1.10
140	3006	4.72	170	1.02	1735	8.17	294	1.04	1002	14.15	509	1.07
160	3926	4.13	149	1.02	2267	7.15	257	1.03	1309	12.38	446	1.06
180	4969	3.67	132	1.01	2869	6.35	229	1.03	1656	11.01	396	1.04

It is clear that there is a difference in lift between the inside wing and the outside wing at the wing tip. Calculus tells us that the average difference in lift across the wing (assuming that the portion of the wing at the fuselage provides the same lift as the outer portion of the wing, which it does not) is $1/3^{rd}$ the lift difference at the tip and this average lift difference is located at just a bit less than 0.6 ℓ.

Say our 36 ft. wingspan aircraft with a weight of 2500 lb. is flying at a TAS of 100 knots in a level 45° turn. There is an average of $1/3^{rd}$ of the 6% wing tip lift difference between the outside and inside wing acting at an average lever arm of about 10 feet (~ 0.6 ℓ) either side of the center of gravity. $1/3^{rd}$ of 6% (2%) of the the average lift difference is 50 lb. (2% of 2500 lb.) acting through a lever arm of 10 feet gives a rolling moment of 500 ft-lb. Not an insignificant amount of roll moment Imagine a couple of your not-so-dainty 250 lb. friends sitting together 10 feet out on one wing and the roll compensation that would be required. You might not find it impossible to imagine that with the plane on the ground and one of those 250-pounders sitting 10 feet out on one wing and the second pushing the opposite wing up with 250 lbs. force (which is how the asymmetric airflow results in a roll moment)) that they might be able to actually tip the airplane over onto one wingtip. Fortunately in flight (again, very design-dependent) it only takes a small amount of aileron (maybe a couple or three degrees, maybe not even that) to counteract the roll. And, if you are in a high-wing aircraft that has a bit of pendulum effect you may not need any aileron input at all in smooth air.

Looking at the table again you can see that the slower you are and the steeper your bank the more the lift difference. This is because the lift difference is dependent on the turn radius (the smaller the radius the greater the lift difference) and turn radius gets smaller with slower speed for a given bank angle or steeper bank angle for a given speed (or of course, any combination of the two). So at some point you are going to either need to counteract the overbanking tendency or exceed the design positive load factor (a 74.75° bank for a normal category aircraft with no updrafts).

Looks like the FAA is right. But we knew that already, didn't we?

Climbing and descending (coordinated) turns and slipping and skidding turns present a whole different ball of worms (or can of wax, depending on which metaphors you want to mix). The subject of a whole new paper to be written at some point, I'm sure!!

And don't get me started about **The Impossible Turn**!!