Best Angle of Climb, Best Rate of Climb

and Why They Depend on Excess Thrust and Excess Power

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Best angle of climb – Vx is the V-speed symbol. Best angle of climb is defined as the greatest amount of altitude that can be gained over the shortest distance. Or, in reference to the speed Vx, it is the speed at which the greatest altitude gain over a given distance can be achieved. The short-field target speed (usually) and then generally only until the obstacles are cleared.

Best rate of climb – Vy is the V-speed symbol. Best rate of climb is the greatest amount of altitude that can be gained over the shortest amount of time. In relation to Vy, this is the speed at which the greatest altitude gain over a given amount of time can be achieved. Some say this is your "usual" departure climb speed but others argue this may be unnecessarily slow – but that's a different discussion.

Some remember that Vx is the best angle of climb speed because the 'X' has a lot of angles (so does 'Y' but I guess not as many so 'X' wins the angle count). Presumably they remember that Vx and Vy are the 'best *something* climb speeds' so by the process of elimination Vy would be the best rate of climb speed.

Also, I will venture to guess that some, if not many, simply memorize two facts:

1. Best angle of climb depends on the amount of *excess thrust* available.
2. Best rate of climb depends on the amount of *excess power* available.

Here's why that is so:

The energy equation –

Work in (W_i) + Kinetic Energy at the start of the process (KE_1) + Potential Energy at the start of the process (PE_1) is equal to

Kinetic Energy at the end of the process (KE_2) + Potential Energy at the end of the process (PE_2) + Work out (W_O) or …

$W_i + KE_1 + PE_1 = KE_2 + PE_2 + W_O$ or, written in terms of what we think of when flying (engine power, air speed, altitude, etc,):

Work from the engine (W_E) + $\frac{1}{2}\,mV_1^2/g_C + mgH_1/g_C = \frac{1}{2}\,mV_2^2/g_C + mgH_2/g_C$ + Work loss due to drag (W_D)

m is the mass of the aircraft
V is the velocity (true airspeed) of the aircraft
H is the height (altitude) of the aircraft
g is the acceleration of gravity – 32.174 ft/sec^2
g_c is the gravitational constant (used only when non-metric units like feet and pounds mass are used)

In steady speed, level flight the work supplied by the engine is equal to the work lost due to drag (velocity and height do not change so the PE and KE terms cancel each other), so $W_E = W_D$

In the case where velocity is constant but the height (altitude) is changing the KE terms cancel but the PE terms do not, so the equation becomes:

W_E + $^{mgH_1}/_{g_C}$ = $^{mgH_2}/_{g_C}$ + W_D and re-writing this in terms of change of height (ΔH):

(W_E - W_D) = $^{mg}/_{g_C}\,(H_2 - H_1)$ = $^{mg}/_{g_C}\ \Delta H$ If we divide both sides by some arbitrary time interval ΔT:

(W_E - W_D) / ΔT = $^{mg}/_{g_C}\ \Delta H/\Delta T$ Note that work over unit time is power, so this becomes:

(P_E - P_D) = ΔH / ΔT - The change in height over time depends on how much bigger P_E is than P_D (or, how much *excess power* your engine has above the power required to overcome drag). It also stands to reason that the greatest excess power would be at full throttle AND at the speed that has the *lowest* power required to overcome drag at that speed and throttle setting. In other words at Vy and full throttle (and max RPM or the appropriate prop speed as specified by the POH for aircraft with an adjustable pitch prop).

Power (P) = Thrust (T) x Velocity (V), so power divided by velocity equals thrust. Dividing both sides of the equation (P_E - P_D) = ΔH / ΔT by V you get (P_E - P_D)/V = ΔH / (VΔT) or T_E - T_D = ΔH / (VΔT); and velocity flown for some interval of time ΔT would be the interval of distance covered ΔS, so –

T_E - T_D = ΔH / ΔS, the change in height (altitude) over an interval of distance, so the greatest change in height for a given interval of distance would be at the speed (Vx) and throttle setting (maximum throttle, prop set accordingly if pitch is adjustable) where the difference between the thrust produced by the engine and the thrust required to overcome drag is greatest - the point of maximum *excess thrust*.

The energy equation shows why Vx is dependent on the excess thrust available and Vy is dependent on the excess power available. BUT – for a propeller driven aircraft thrust is produced by the action of the propeller and the power of the engine (not directly, as with a jet engine-powered aircraft). So the point (or speed) of maximum excess power (and, therefore, Vy) is NOT necessarily the point of the lowest power needed to overcome the work of drag. It is the point where the difference between power available and power required is the greatest. And, the point where the greatest excess thrust is available (at speed Vx) does not necessarily correspond with the lowest amount of drag that can be achieved but rather where the difference of thrust available and thrust required are greatest. The point (speed) of the lowest total drag (D) is $(L/D)_{MAX}$ but for a propeller-driven aircraft this is not necessarily (and usually is not the same) as either Vx or Vy – Boldmethod explains this well in **Why Vx and Vy are different**. Note that the thrust available may not be a simple down-sloping line as shown in their examples but for a propeller-driven aircraft it is NOT a horizontal line (i.e., thrust is not constant at all velocities).

For a jet aircraft (a ‘thrust aircraft’) thrust IS constant at all speeds, so Vx IS the same speed as the speed at $(L/D)_{MAX}$

Propeller and jet aircraft have one thing in common: Take away all power and thrust their best glide speed (generally determined at max gross weight) is at $(L/D)_{MAX}$. But best glide speed is not really speed – it is the best glide ANGLE – see the answer to **Question 8 in The Aerodynamics of Other Stuff** on the web site.